

# Quantum Cryptography

Bertrand Bonnefoy-Claudet    Zachary Estrada

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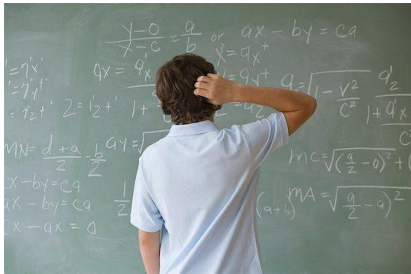
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**Modern crypto relies on computational difficulty**



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- AES 256 as strong as AES 128 w/conventional
- RSA&Co solved in polynomial time (Shor's algorithm)
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**What is secure?**

# Proven security

Secure cipher

Cyphertext gives zero knowledge about message or key

Required properties of a key:

- as long as message
- random
- used only once

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Modulo-2 addition (or XOR):

Message: 0000110011110010

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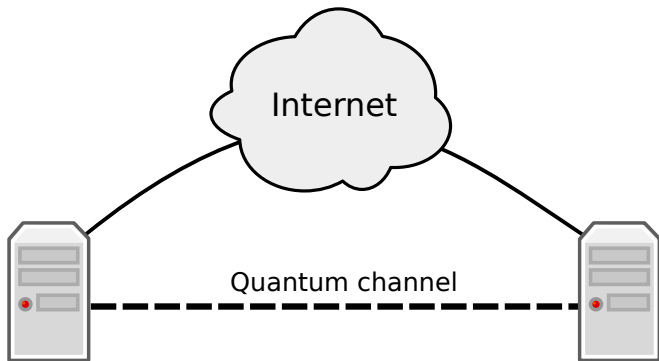
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**Let's rely on nature**

# Quantum channel



# Key exchange

- Qubits cannot be measured without being modified
- Key exchange protocol over quantum channel
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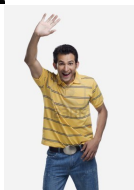
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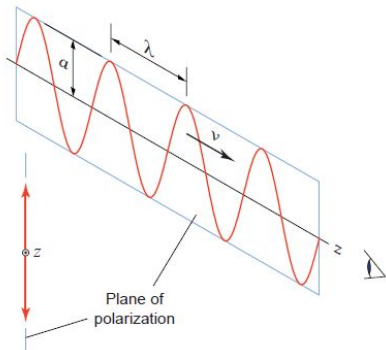




# Classical Waves

# Polarization

Simplified wave:  $\vec{E}$  field 'pointing' in an alternating directions like a moving sine function.

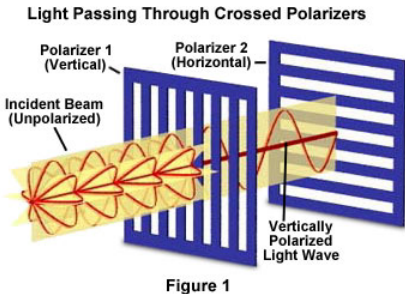
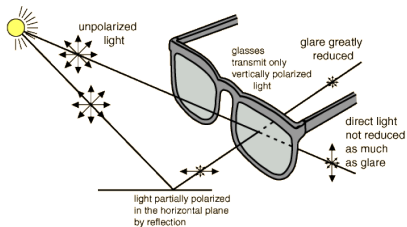


Yes, this started out as a lowres gif



# Polarization (cont.)

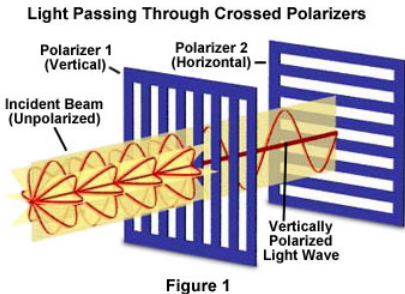
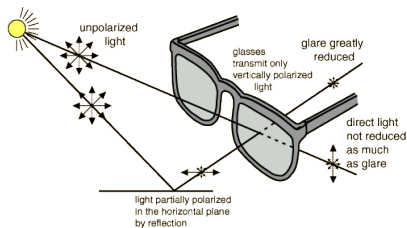
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We will see that you can also to encode information in polarization states

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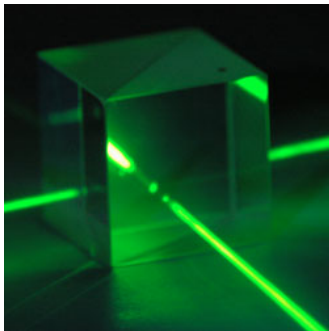
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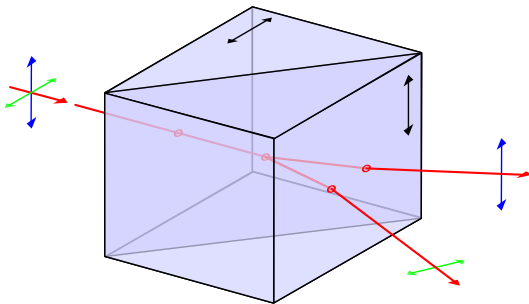
# Beamsplitters

Earlier in lecture we've talked about 'half-silvered mirrors' or beamsplitters



## Beamsplitters (cont.)

By using the right combination of materials, a beamsplitter can split on polarization state:



We call this a Polarizing BeamSplitter (PBS)

# Quantum Mechanics

I think I can safely say that nobody understands quantum mechanics.

–Richard Feynman

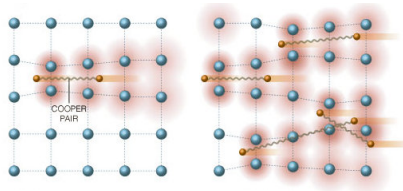


# Quantum Mechanics

Quantum Mechanics has been used to explain many things

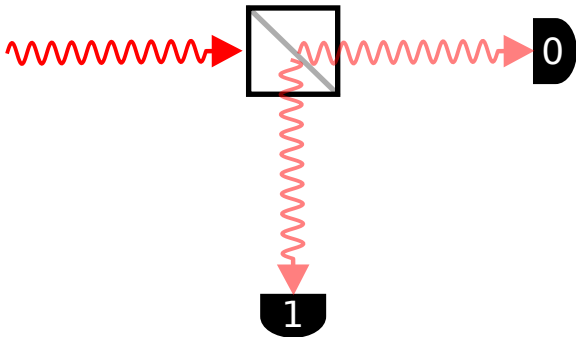
Periodic Table of the Elements

1	2											18	18	18	18	18	18	18	18																																																																																																				
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H	He											B	C	N	O	F	Ne																																																																																																						
Li	Be											Al	Si	P	S	Cl	Ar																																																																																																						
Na	Mg											K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr																																																																																										
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Actinide Series		Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr																																																																																																							
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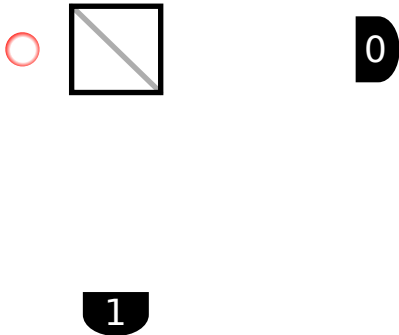
## QM example - photon counting

- Consider a monochromatic light source, a beam splitter, and two detectors



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- So we saw two waves with half-intensity. What happens for a single photon?



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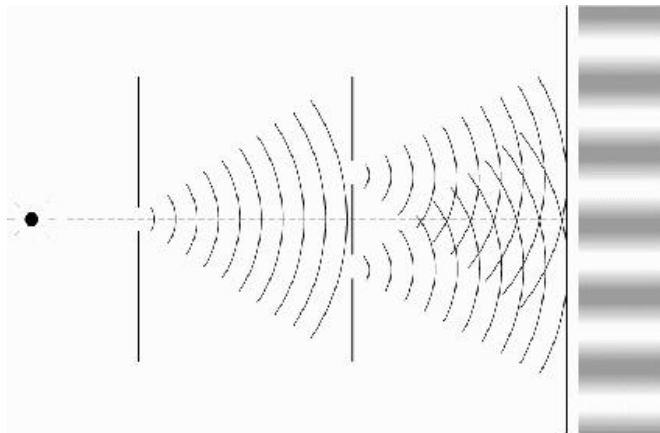
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There are no “half-photons.” Given a perfect beamsplitter, each detector clicks half of the time. There is no way for us to predict which way it will go.

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## QM example - double slit



<http://www.toutestquantique.fr/#dualite>

# Superposition

## Quantum Superposition

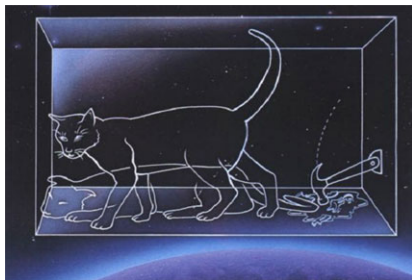
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# Uncertainty

## Probability and Uncertainty in Nature

At the quantum scale, it is *impossible* to predict the *exact* outcome of certain events. Furthermore, certain quantities are *fundamentally unknowable*.

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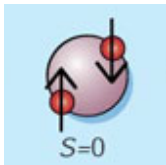


# Entanglement - an example

## Pauli Exclusion Principle

Two electrons cannot occupy the exact same state

- Consider two electrons in a helium atom, in the lowest energy state ( $1s^2$ , if you remember chemistry) with spin 0.
- This means that one  $e^-$  is  $\uparrow$  and the other is  $\downarrow$
- If we look at one  $e^-$ 's spin, we immediately know the other's

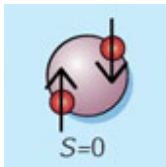


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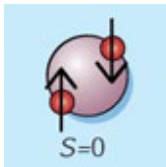


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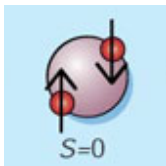


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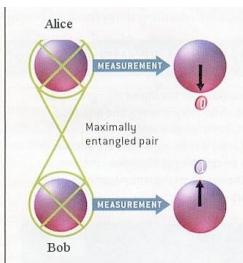
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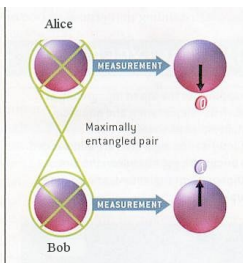


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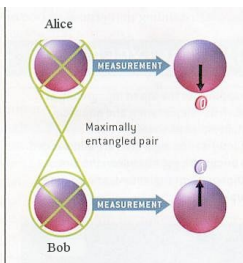
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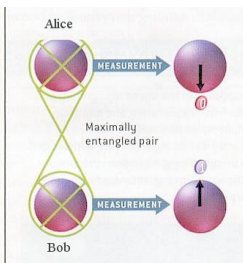
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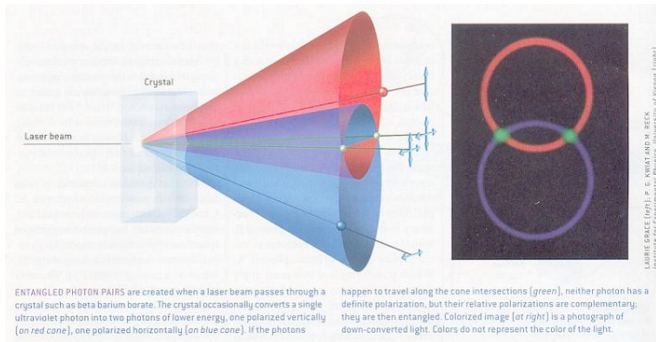
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# Making entangled photons

BBO: Spontaneous parametric down-conversion converts one photon into two photons



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In 2012, 144km



## QM - wrapup

From Gisin *et al.*, Quantum Cryptography (2002):

- One cannot take a measurement without perturbing the system.
- One cannot determine simultaneously the position and the momentum of a particle with arbitrarily high accuracy.
- One cannot simultaneously measure the polarization of a photon in the vertical-horizontal basis and simultaneously in the diagonal basis.
- One cannot draw pictures of individual quantum processes. (You can only measure observables)
- One cannot duplicate an unknown quantum state.



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# Quantum Cryptography

(or Quantum Key Distribution)

# Goal

**Exchange a key**

# Example: BB84

Let's start with an example: BB84

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- Originally used polarization

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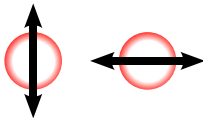
# Polarization

Many kinds of polarization:



# Bases

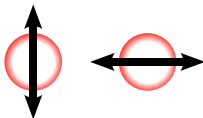
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- Components are linearly independent
- From this basis and **superposition**, you can get all other states

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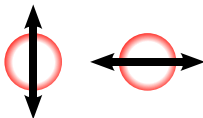
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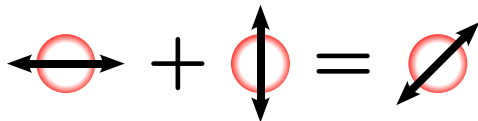
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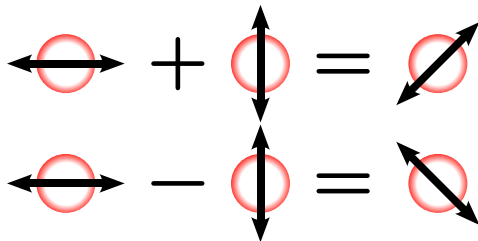


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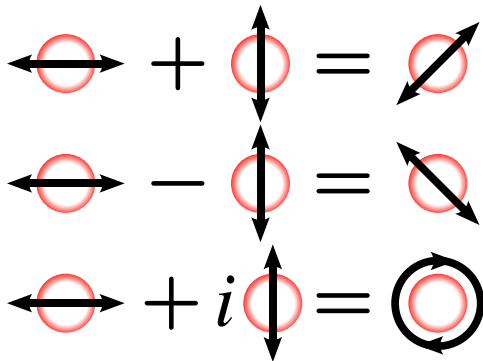
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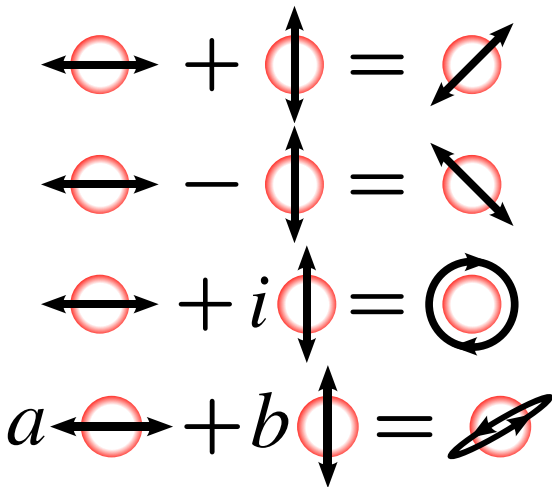


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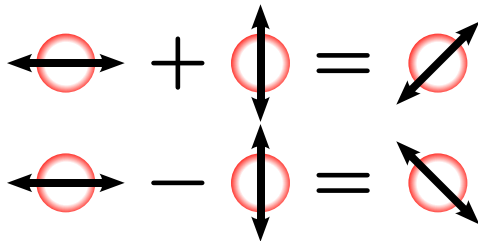




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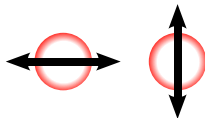
We'll focus on these.

# Two bases

- Horizontal – Vertical
- Diagonal – Antidiagonal

## Two bases

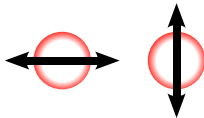
- Horizontal – Vertical



- Diagonal – Antidiagonal

## Two bases

- Horizontal – Vertical

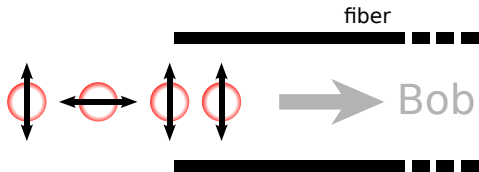


- Diagonal – Antidiagonal



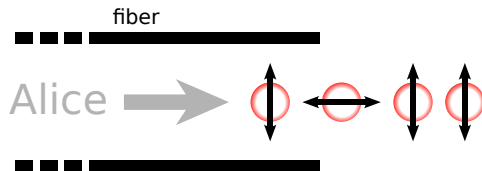
# Sending bits

Alice sends 1101:

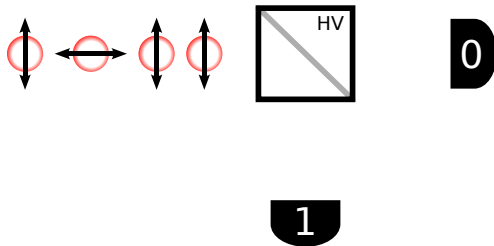


# Receiving bits

Bob receives. . .



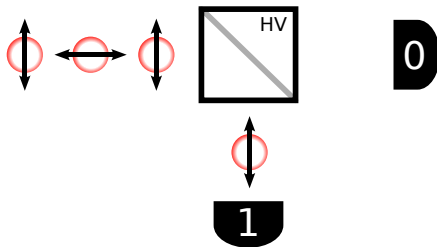
# Receiving bits



Bob gets:

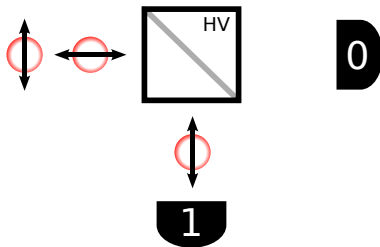


# Receiving bits



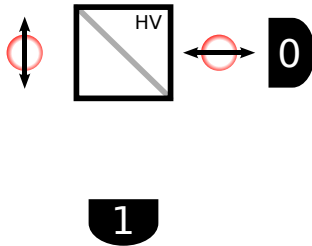
Bob gets: 1

# Receiving bits



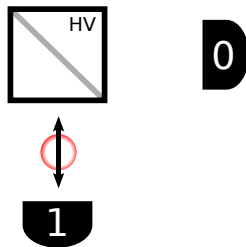
Bob gets: 11

# Receiving bits



Bob gets: 110

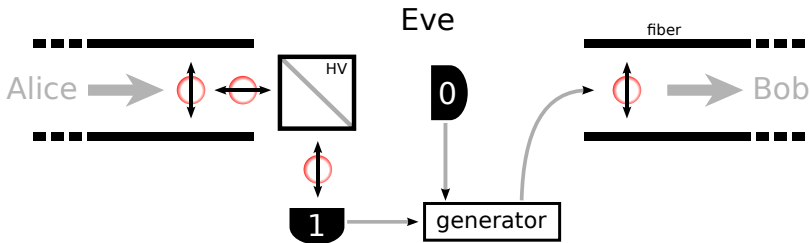
# Receiving bits







Bob gets: 1101

# Eavesdropping

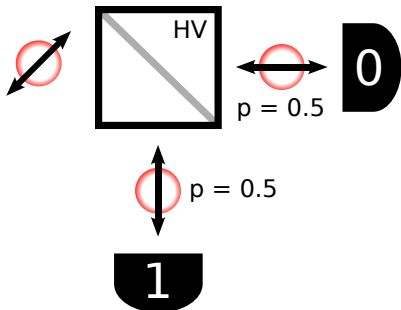
Basis is known, so Eve can measure and regenerate:



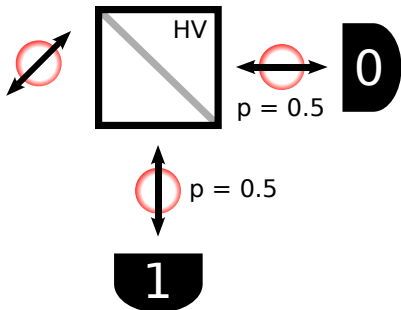
# Choose bases randomly

Message	1	1	0	1
Basis	HV	DA	DA	HV
Polarization				

# Measurement with wrong basis



## Measurement with wrong basis

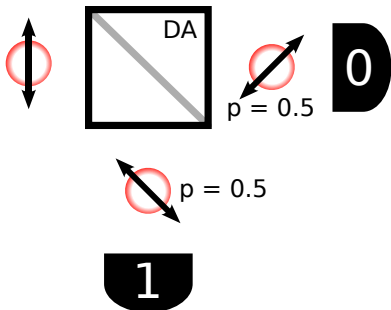


It comes from superposition:

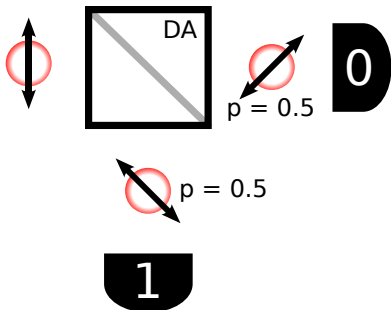
The equation shows the superposition of two bases. On the left, a red circle with a horizontal arrow pointing from left to right. This is followed by a plus sign, then a red circle with a vertical arrow pointing from top to bottom. This is followed by an equals sign, and finally a red circle with a diagonal arrow pointing from the bottom-left to the top-right.



# Measurement with wrong basis



# Measurement with wrong basis



Superposition, too:



# Measurement with wrong basis

Using the wrong basis implies:

- measurement unreliability
- quantum state perturbation

# Measurement with wrong basis

Using the wrong basis implies:

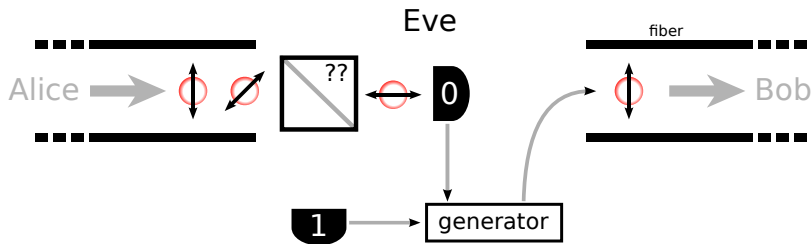
- measurement unreliability
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Using the wrong basis implies:

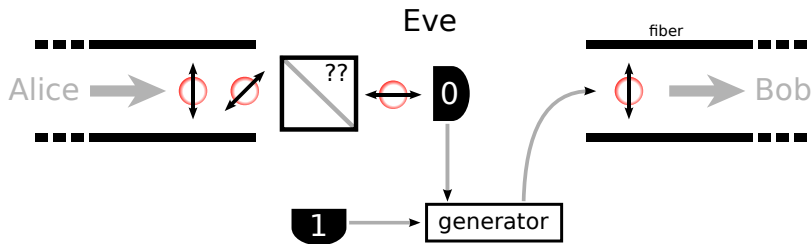
- measurement unreliability
- quantum state perturbation

# Eavesdropping: fail



















- Not obvious, what about Bob?

# Eavesdropping: fail



















- Not obvious, what about Bob?

# Agreement

















Alice	1	1	0	1	1	1	0	1
	HV	DA	DA	HV	DA	DA	HV	HV
								
Bob	HV	DA	HV	HV	HV	DA	HV	DA
								
	1	1	0	1	0	1	0	0



















# Agreement

	1	1		1	1	1	0	1
Alice	HV	DA	DA	HV	DA	DA	HV	HV
								
Bob	HV	DA	HV	HV	HV	DA	HV	DA
								
	1	1		1	0	1	0	0

















# Agreement

	1	1		1		1	0	1
Alice	HV	DA	DA	HV	DA	DA	HV	HV
								
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	1	1		1		1	0	0

# Agreement

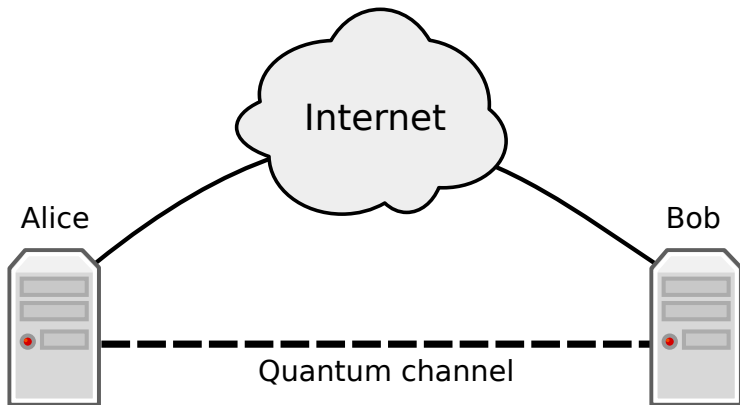
	1	1		1		1	0	
Alice	HV	DA	DA	HV	DA	DA	HV	HV
								
Bob	HV	DA	HV	HV	HV	DA	HV	DA
								
	1	1		1		1	0	

# Agreement

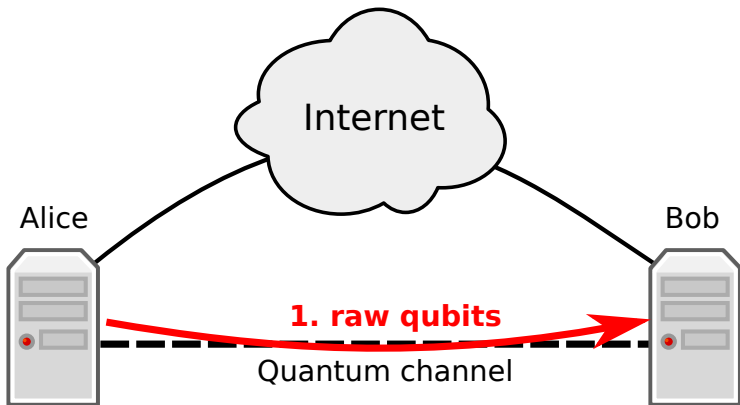
Alice	1	1		1		1	0	
	HV	DA	DA	HV	DA	DA	HV	HV
								
Bob	HV	DA		HV	HV	DA	HV	DA
								
	1	1		1		1	0	

They end up with the same bits, called a sifted key

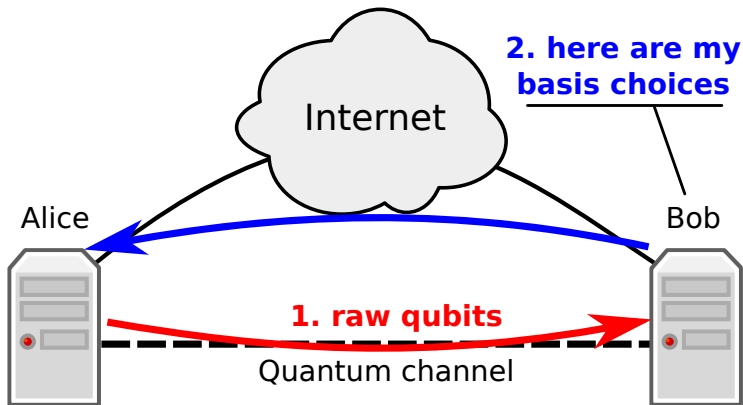
# Agreement



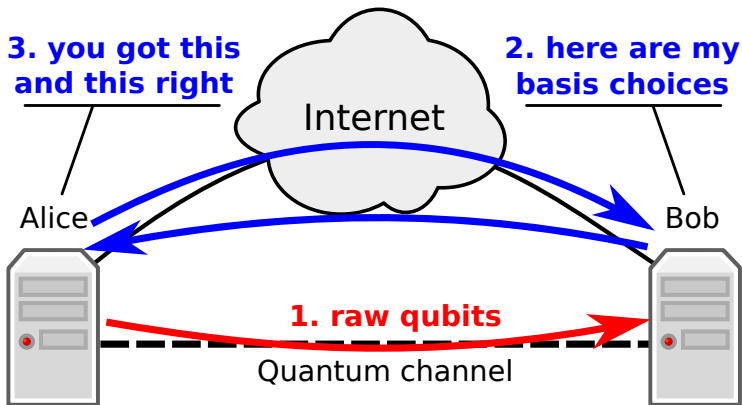
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# Secure?

- Messages must be authenticated
- Alice and Bob lose 50% of the raw bits on average
- Eve can get some information from bits and messages
- How much?
  - 75% of correct bits (but she wouldn't necessarily know which ones)
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- Can she be detected?

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

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  - More info with messages
- Can she be detected?



















## Agreement and sacrifice

	1	1	0	1	1	1	0	1
Alice	HV	DA	DA	HV	DA	DA	HV	HV
								
Bob	HV	DA	HV	HV	HV	DA	HV	DA
								
	1	1	0	1	0	1	0	0

## Agreement and sacrifice

	1	1		1	1	1	0	1
Alice	HV	DA	DA	HV	DA	DA	HV	HV
								
Bob	HV	DA	HV	HV	HV	DA	HV	DA
								
	1	1		1	0	1	0	0

## Agreement and sacrifice

Alice	1	1		1		1	0	1
	HV	DA	DA	HV	DA	DA	HV	HV
								
Bob	HV	DA	HV	HV	HV	DA	HV	DA
								
	1	1		1		1	0	0

















# Agreement and sacrifice

	1	1		1		1	0	
Alice	HV	DA	DA	HV	DA	DA	HV	HV
Bob	HV	DA	HV	HV	HV	DA	HV	DA
	1	1		1		1	0	

# Agreement and sacrifice

		1		1		1		0
Alice	HV	DA	DA	HV	DA	DA	HV	HV
Bob	HV	DA	HV	HV	HV	DA	HV	DA
		1		1		1		0

# Agreement and sacrifice

























		1			1	0		
Alice	HV	DA	DA	HV	DA	DA	HV	HV
								
Bob	HV	DA	HV	HV	HV	DA	HV	DA
								
		1			1	0		

# Agreement and sacrifice

		1			1	0		
Alice	HV	DA	DA	HV	DA	DA	HV	HV
Bob	HV	DA	HV	HV	HV	DA	HV	DA
		1			1	0		

























Smaller sifted key

# Eavesdropping defeated

























Alice	1	1	0	1	1	1	0	1
	HV	DA	DA	HV	DA	DA	HV	HV
								
Eve	HV	HV	DA	DA	DA	DA	HV	DA
								
	1	0	0	1	1	1	0	0
Bob	HV	DA	HV	HV	HV	DA	HV	DA
								
	1	1	0	0	0	1	0	0



























# Eavesdropping defeated

Alice	1	1		1	1	1	0	1
	HV	DA	DA	HV	DA	DA	HV	HV
								
Eve	HV	HV	DA	DA	DA	DA	HV	DA
								
	1	0	0	1	1	1	0	0
Bob	HV	DA	HV	HV	HV	DA	HV	DA
								
	1	1		0	0	1	0	0

# Eavesdropping defeated

	1	1	1	1	1	0	1	
Alice	HV	DA	DA	HV	DA	DA	HV	HV
								
Eve	HV	HV	DA	DA	DA	DA	HV	DA
								
	1	0	0	1	1	1	0	0
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	1	1	0	0	1	0	0	

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Alice	1	1		1		1	0	
	HV	DA	DA	HV	DA	DA	HV	HV
								
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	1	0	0	1	1	1	0	0
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	1	1		0		1	0	

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Alice		1		1		1	0	
	HV	DA	DA	HV	DA	DA	HV	HV
Eve								
	HV	HV	DA	DA	DA	DA	HV	DA
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	1	0	0	1	1	1	0	0
	HV	DA	HV	HV	HV	DA	HV	DA
		1		0		1	0	

The diagram illustrates a quantum communication protocol where Alice and Bob share a secret key by comparing their measurement results. Eve's eavesdropping attempts are detected because her measurements introduce errors in the key. The key bits are 1, 0, 1, 1, 1, 0, 0.

# Eavesdropping defeated

Alice		1		1		1	0	
	HV	DA	DA	HV	DA	DA	HV	HV
Eve								
	HV	HV	DA	DA	DA	DA	HV	DA
Bob								
	HV	DA	HV	HV	HV	DA	HV	DA
		1		0		1	0	
		1		0		1	0	

**Error detected**

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Where can they come from?

- Loss (fiber, filters, etc)
- Polarization perturbation
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- BB84: 4 quantum states and random basis choices
- B92: Similar but with 2 quantum states
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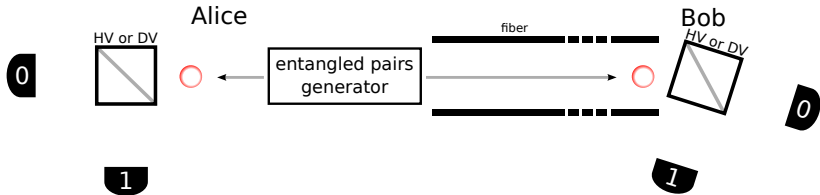


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- Perform several rounds
- Key should grow at each round

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- Messages authenticated with **part of the initial key**
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## Breaking QKD



QKD is theoretically proven to be secure, but is there a large gap between ideal theory and actual implementations? What about side channels?

# Fake states

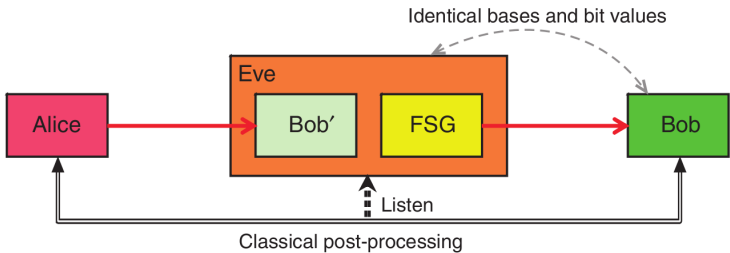
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# Fake State Generator

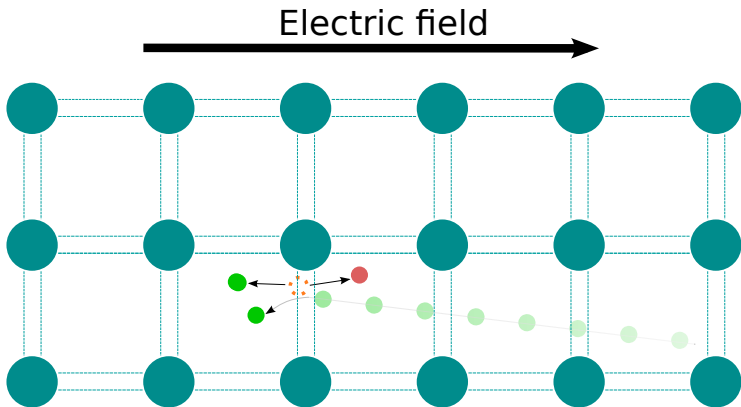


# The attack

- Used an actual QKD system (E91 protocol) from previous experiments
- Inserted Eve into a  $\sim 300$  m setup
- Eve uses identical measuring equipment
- Eve also forces Bob's polarization basis choice
- Again, the Quantum parts are still valid and secure

# A quick background on detectors

Impact ionization



## A quick background on detectors (cont.)

- Impact ionization in an area with a high electric field can lead to an “avalanche current”
- An external circuit is used to then quench the avalanche current and then recharge the circuit
- Main idea: a single photon is enough to cause a macroscopic current because of the avalanche process

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## Faking states

If Eve measures the photons, could we use classical light instead of single photons to control the detector?

- We wouldn't have asked if the answer wasn't yes!
- There is a stray capacitance that needs to recharge for the next avalanche
- If enough photons keep hitting the diode so that the cap can't recharge, the avalanche current decreases (c.w.)
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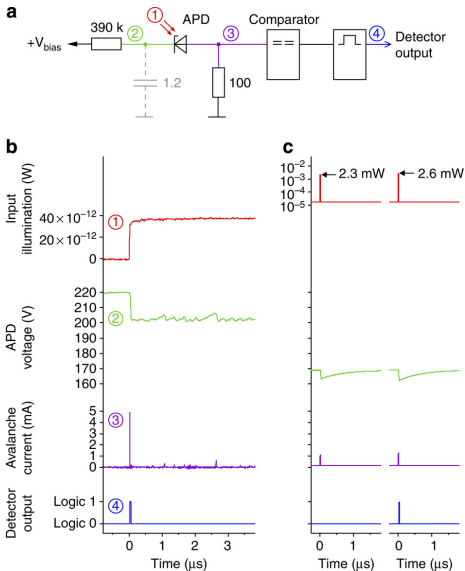
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# Faking states (cont.)



## Hooray for faked states

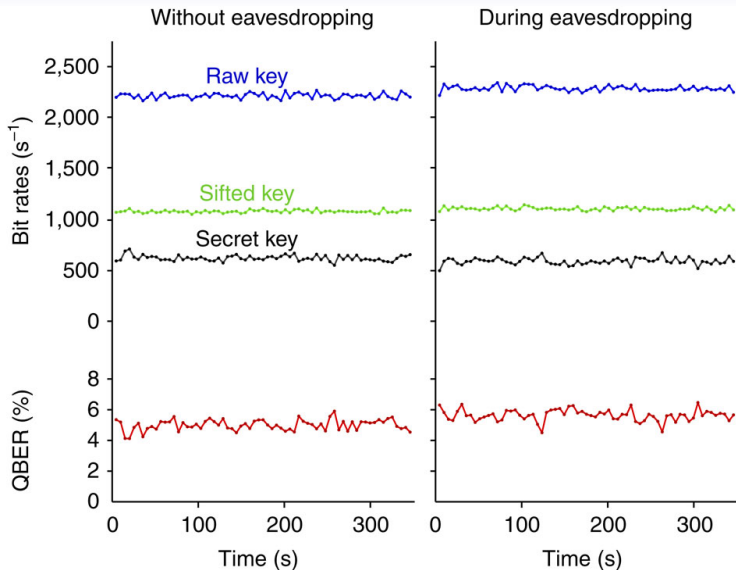
Faked states sent		Clicks at Bob's Detector			
		V	A	H	D
1,702,067	V	1,693,799 (99.51%)	0	0	0
2,055,059	A	0	2,048,072 (99.66%)	0	0
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**The wrong detector is NEVER triggered!**

# Hooray for faked states (cont.)



## Some thoughts on this attack

- It is based on a specific implementation
- Requires passive basis choice, but...
- Could be detected by measuring intensity
- Brings up a good point: Does the security of QKD actually rely on nature? Or just how well we can build systems?

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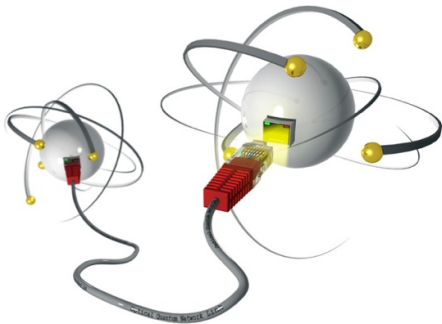
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# Quantum Networks



# What are they?

A quantum network is a set of quantum nodes connected by quantum channels

Main motivations for building quantum networks:

- Connecting quantum computing/communication elements
- Investigating quantum interactions (fundamental research)

This can be achieved by sending quantum particles or distributing entanglement interactions.

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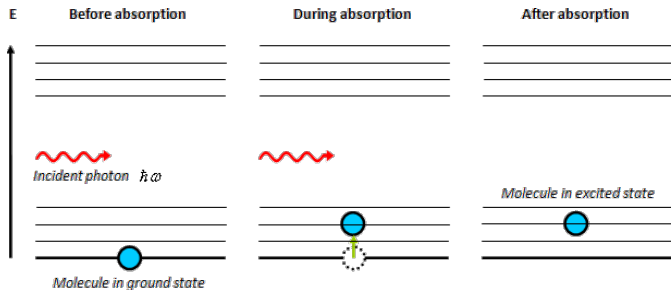
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For a single photon, probability of absorption  $\sim \exp(-L_{\text{fiber}})$

## The problem (cont.)

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Introducing Quantum Repeaters:

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- At these connection points, we can swap entanglement states
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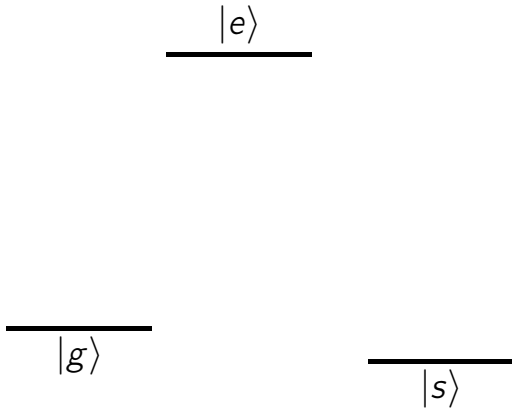
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## A little bit more on atoms and photons



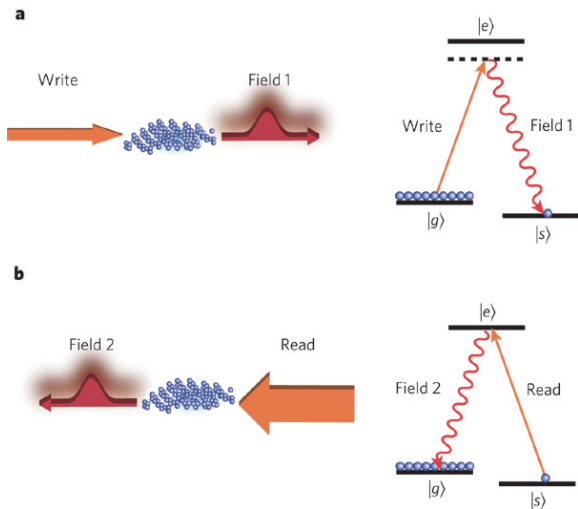
## State transfer using atomic energy levels

- If we can force a transition from  $|g\rangle \rightarrow |e\rangle \rightarrow |s\rangle$ , then detection of the photon from the  $|e\rangle \rightarrow |s\rangle$  transition can herald our storage
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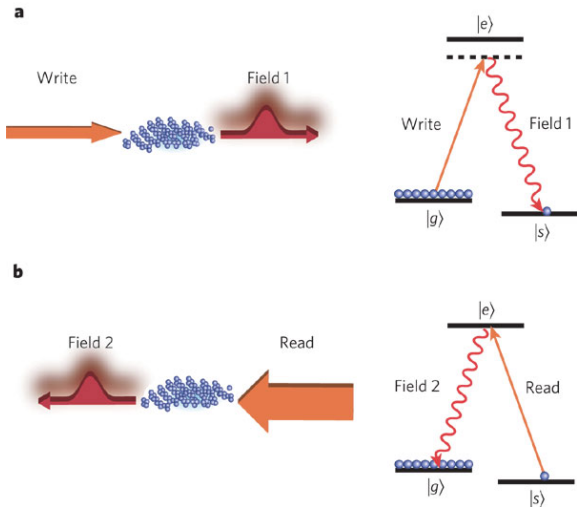
- If we can force a transition from  $|g\rangle \rightarrow |e\rangle \rightarrow |s\rangle$ , then detection of the photon from the  $|e\rangle \rightarrow |s\rangle$  transition can herald our storage
- However, ensuring a particular photon couples with a specific atom is difficult for many reasons
- So use lots of photons and lots of atoms!

# State transfer using many-body systems (picture form)



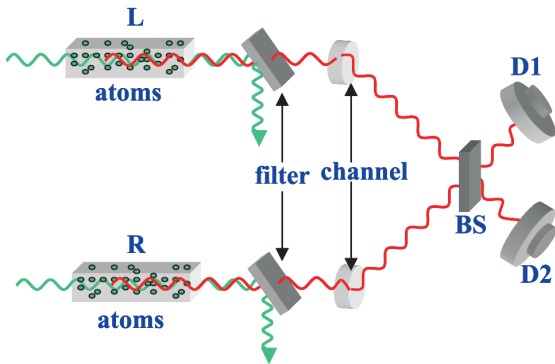
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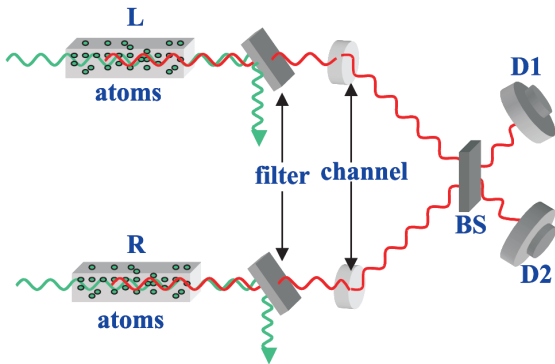
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# Duan, Lukin, Cirac and Zoller (DLCZ) Protocol



- The pulses from the photons interfere at the 50/50 NPBS
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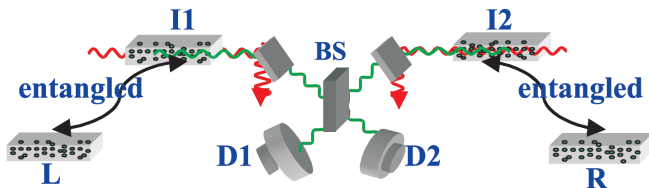
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- A single click indicates that one of the ensembles (we don't know which) has transitioned from  $|g\rangle$  to  $|s\rangle$



# DLCZ Repeater



- Prepare two entangled pairs
- “Read” the states simultaneously
- Just like before, the photons interfere at the BS and a click signals success (L & R are entangled)
- This allows for quantum communication over long distances

## DLCZ Repeater thoughts





- Can tolerate certain inefficiencies very well - photon detectors  
50% or lower efficiency should work
- However, this is still a highly intricate system
- But, the error rate is projected to be  $\ll$  than the attenuation
- Still waiting on some good experiments








# The Grand Conclusion

- QKD is theoretically secure and appears to be feasible (with existing commercial implementations)
- As always, implementation is a key detail regarding security
- Quantum networks are a long, long way off
- Research on quantum computing seems to be worse off than communication, so you've still got time left on your private keys





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